

Enhancement and Resolution of Singularity

Satoshi Yamaguchi¹

*Graduate School of Human and Environmental Studies,
Kyoto University, Kyoto, 606-8501, Japan*

Abstract

We review the enhancement mechanism proposed by Johnson, Peet and Polchinski. If we consider the D6-brane wrapped on K3, then there appears a naked singularity called “repulson” in the supergravity solution. But this singularity is resolved by a shell structure called “enhancement”. In the interior of the enhancement, the abelian gauge symmetry is enhanced to a nonabelian one, and ordinary supergravity is no more reliable. We also review the interpretation of enhancement as fuzzy sphere. This paper is the contribution to the proceedings of “Frontier of Cosmology and Gravitation”, April 25-27 2001, YITP.

1 Introduction

The string theory includes the graviton in its massless spectrum, and each order of the loop expansion is finite. Therefore, the string theory is thought as the “quantum theory of the gravity” (For a review of the string theory, see [1]). However, the formulation of the string theory depends on background, and we can only formulate it as a fluctuation from the background.

Let us consider what happens when the background has some geometrical singularity. If one can formulate the string theory consistently on the geometrically singular background, let us call it “resolution of singularity in the string theory”. For a review of singularities in the string theory, see [2].

Orbifolds are the most famous examples of resolution of singularity in string theory. The string theory on an orbifold can be formulated consistently by including the twisted sectors. The twisted sectors correspond to the resolution modes.

Also D-branes are interesting objects from the point of view of singularities. A D-brane is the object on which open string can end [1]. On the other hand, a D-brane can also be described as a solution of the classical supergravity [3] — the low energy effective theory of the string theory. The supergravity description and the open string description of a D-brane do not look very similar at first sight. The evidences of the equivalence of these two are as follows.

- The supergravity description and the open string description of a D-brane have the same charge called Ramond-Ramond charge.
- Both of them are BPS saturated states. BPS saturated states are the lowest energy states out of those which have the same charge, that is, extremal in terms of the relativity.

For this reason, we regard them as the same object.

If we describe this D-brane as a solution of the supergravity, there often appear naked singularities. But, D-branes are not “bad” objects in open string description, the singularities in supergravity description should be resolved by some mechanisms.

The singularity treated in this paper — repulson singularity — is one of the D-brane singularities [4]. It does not resemble to ordinary D-brane singularities in the supergravity description. However, we know that the string theory on this D-brane background is not singular, therefore, the geometric singularity is resolved. We intend to see how this singularity is resolved from the geometrical point of view and clarify

¹E-mail:yamaguch@phys.h.kyoto-u.ac.jp

what happens near the singularity. In this point of view, Johnson, Peet and Polchinski [4] propose that there appears a special radius called “enhançon radius” and D-branes form a “shell” at this radius. And in inner region of this radius, the geometry seems to be flat and there are no singularity. We review this “enhançon mechanism” in this paper.

The organization of this paper is as follows. In section 2, we consider the D-brane configuration called “D6-D2* system” and see how the singularity appears. We also analyze this system by the D-brane probe and see a special radius called “enhançon radius” appears. In section 3 we interpret the enhançon in terms of the gauge theory and explain the proposal of the “shell”. In section 2, we introduce another point of view of the enhançon shell as a fuzzy sphere.

2 D6-D2* system

Let us consider the Type IIA string theory on $\mathbb{R}^{5,1} \times K3$. Here, the manifold “K3” is a 4-dimensional Ricci flat Kähler simply connected compact manifold. The manifolds which satisfy the above conditions are all diffeomorphic. For a review of string theory on K3 see [5].

We consider N D6-branes wrapped on whole K3 in the above theory. In the view of the 6-dimensional theory, these D6-branes look 2-branes. This system has not only N D6-brane charge but also $(-N)$ D2-brane charge [6] of shown in table 1. This is because K3 is curved and the stringy correction arises. Actually, these negatively charged D2-branes are not the ordinary anti-D2-branes and have negative contribution to the brane tension. We call this system “D6-D2* system”. This D6-D2* system is BPS saturated and conserves 8 supercharges.

	0	1	2	3	4	5	6	7	8	9
	$\mathbb{R}^{5,1}$						K3			
D6	○	×	×	×	○	○	○	○	○	○
D2*	○	×	×	×	○	○	×	×	×	×

Table 1: D6-D2* system. The branes spread to the direction marked by ○, and do not spread to the direction marked by ×. There are actually no D2-brane, but we write D2-brane in this table in order to express the D2-brane charge.

2.1 Classical solution

The low energy effective theory of the type IIA string theory is the type IIA supergravity. The type IIA supergravity includes the following fields as bosonic fields.

- NSNS fields — g_{MN} (metric), $B^{(2)}$ (NSNS 2-form), Φ (dilaton).
- RR fields — $C^{(1)}$ (RR 1-form), $C^{(3)}$ (RR 3-form).

A D6-brane has magnetic charge of the gauge field $C^{(1)}$. It is convenient to denote the electro-magnetic dual of $C^{(1)}$ by 7-form $C^{(7)}$. Then, A D6-brane has electric charge of $C^{(7)}$. On the other hand, D2-brane has electric charge of $C^{(3)}$. Sometimes it is convenient to use 5-form $C^{(5)}$, which is the electro-magnetic dual of $C^{(3)}$.

Now we write down the classical solution of D6-D2* system. Let the longitudinal direction on the brane $\mu, \nu = 0, 4, 5$, and their flat metric $\eta_{\mu\nu} = \text{diag}(-1, 1, 1)$. We denote the transverse direction of the branes as $i, j = 1, 2, 3$. Let us also define the radial coordinate $r^2 = x^i x^i$. By using these notations, the solution can be written down as

$$\begin{aligned}
ds^2 &= Z_2^{-1/2} Z_6^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + Z_2^{1/2} Z_6^{1/2} dx^i dx^i + Z_2^{1/2} Z_6^{-1/2} V_0^{1/2} ds_{K3}^2, \\
e^{2\Phi} &= g_0^2 Z_2^{1/2} Z_6^{-3/2}, \\
C^{(3)} &= (Z_2 g_0)^{-1} dx^0 \wedge dx^4 \wedge dx^5, \\
C^{(7)} &= (Z_6 g_0)^{-1} dx^0 \wedge dx^4 \wedge dx^5 \wedge (\text{vol}(K3)).
\end{aligned}$$

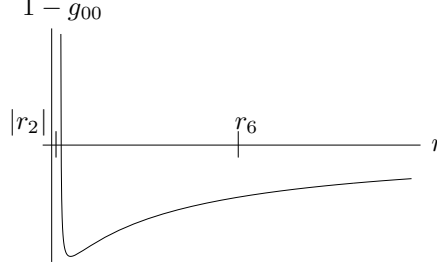


Figure 1: The potential of D6-D2* system. At $r = |r_2|$ the potential diverge. Near the singularity, there is repulsive force, and At far from the singularity, there is attractive force.

Here, we use the harmonic function Z_2 and Z_6 of the form

$$Z_2 = 1 - \frac{|r_2|}{r}, \quad r_2 = -\frac{(2\pi)^4 g_0 N \ell_s^5}{2V_0}, \quad (1)$$

$$Z_6 = 1 + \frac{r_6}{r}, \quad r_6 = \frac{g_0 N \ell_s}{2}. \quad (2)$$

In the above equations, ds_{K3}^2 is a Ricci flat metric on K3 of unit volume, and $\text{vol}(K3)$ is the volume form of the K3, which is 4-form. We also used here the constant ℓ_s called “string length”. The string length characterizes length scale of the stringy effects. If one want to see smaller things than ℓ_s or higher energy things than $1/\ell_s$, he should use string theory rather than supergravity.

In the above solution, there are two continuous parameter (integration constant) g_0 , V_0 and one discrete parameter N . The g_0 is the string coupling constant at far from the branes. The V_0 is the volume of the K3 at far from the branes.

Let us consider the parameter region where the classical supergravity is valid at least far from the brane. First, to suppress the loop contribution of the string theory, g_0 should be small. We also suppose V_0 is very smaller than the stringy volume $V_* := (2\pi\ell_s)^4$ in order to avoid the stringy effects. Then, we can see from (1) and (2) that $|r_2|/r_6 = V_*/V_0$ is very smaller than 1, so we can conclude $|r_2| \ll r_6$ when we suppose the supergravity is valid at infinity. This means that if we observe this system far from the brane, we cannot see the effect of $D2^*$ charge.

Now, we consider the singularity. The point $r = |r_2|$ is a naked singularity. This singularity is “repulsive” which mean there are repulsive force near the singularity. Therefore, this singularity is called “repulsion singularity”. In figure 1, we show the gravitational potential.

Let us remark the validity of the supergravity near the singularity. The singular point is $r = |r_2| = \frac{1}{2} \left(\frac{V_*}{V_0} \right) g_0 N$. The validity of the supergravity at infinity requires both $\left(\frac{V_*}{V_0} \right)$ and g_0 small. If N is not so large, $|r_2| \ll \ell_s$ and supergravity or geometrical consideration is no more valid. The region of repulsive force is the same order as $|r_2|$. Therefore, If we intend to view the repulsion singularity by supergravity point of view, we should set N very large in order to $|r_2| \gg \ell_s$. However, even if we set N very large, the compactification of IIA supergravity is no more valid, because the volume of the K3 $V := V_0 Z_2/Z_6$ becomes arbitrary small near the singularity. Especially, there is the point $V = V_* = (2\pi\ell_s)^4$, which is called “enhanced radius” in the following context. At this point, extra massless fields appear as we see later.

In the following subsection, we investigate what happens near the singularity.

2.2 Probe analysis

In order to see the nature of the singularity, we use some object as a probe and move it close to singularity. A neutral object cannot reach the singularity because of the repulsive gravity force. Then, how about a charged object? Especially, when we construct D6-D2* system, we gather N of single D6-D2* at a point. Actually, D6-D2*'s are BPS even if they are separate, there are no force between them. For these reasons, we use a D6-D2* as the probe. Do not confuse the source D6-D2* system, and the probe D6-D2*. The

former is composed of N D6-D2*'s and makes the background. The latter is a single D6-D2* used to study the geometry made by the source D6-D2* system.

Let us consider the D-brane dynamics on some nontrivial background. For example, the action of a particle in the electromagnetic field is

$$S = - \int dt m \sqrt{-g} + \int e A^{(1)},$$

where g is induced metric, $A^{(1)}$ is pull-back the electromagnetic field to world-line, e is the charge of the particle, and m is the mass of the particle. On the analogy of a particle, a p -brane action on $(p+1)$ -form field $C^{(p+1)}$ background can be written as follows. First, change the integration region to the $(p+1)$ -dimensional world volume. Next, change the length of the world line to the volume of the world volume. Finally, change the gauge field to $(p+1)$ -form which is naturally integrated on the $(p+1)$ -dimensional world volume. The result is written as

$$S = - \int d^{p+1} \xi T_p \sqrt{-\det g} + \int \mu C^{(p+1)},$$

where T_p is the brane tension, and μ is the brane charge.

In the case of the D6-D2*, the action is a little complicated because of the following reason.

- The D-brane tension depend on the value of dilaton Φ , the tension depend on r . When we integrate in the K3, the tension is also depend on K3 volume.
- As mentioned before, D6-D2* system has negative D2 charge because of the curvature.
- Related above, the curvature of K3 has negative contribution to the tension.

As a result, if we integrate the K3 part, the action becomes

$$S = - \int_M d^3 \xi e^{-\Phi(r)} (\mu_6 V(r) - \mu_2) \sqrt{-\det g} + \mu_6 \int C^{(7)} - \mu_2 \int C^{(3)}, \quad (3)$$

where $V(r) = V_0 Z_2 / Z_6$ as defined before, $\mu_p = (2\pi)^{-p} \ell_s^{-p+1}$ are Dp -brane charges.

Now, we fix the reparametrization symmetry and assume some anzats.

- Static gauge. $x^0 = \xi^0$, $x^4 = \xi^1$, $x^5 = \xi^2$
- The probe move only to r -direction. We also assume r is depend only on ξ^0 .
- We consider the slow motion, and expand by the velocity $\dot{r} := \partial r / \partial \xi^0$.

Then, the action (3) can be rewritten as

$$S = \int d\xi^1 d\xi^2 \int d\xi^0 \left[(\text{const}) + \frac{\mu_6 Z_6}{2g_0} (V(r) - V_*) \dot{r}^2 + \dots \right]. \quad (4)$$

The 0-th order term of the \dot{r} expansion of integrand of action is the potential term. In the action (4), the potential is constant and independent of r . Thus, if the probe is not move ($\dot{r} = 0$) then there is no force, and this system is stable. This is consistent with the fact that this system (the source D6-D2* system and the probe D6-D2*) is BPS if the probe stops.

The second term is the leading term of the kinetic energy. We can see the tension depends on r , and when the volume of the K3 becomes the stringy volume: $V(r) = V_*$, the tension vanishes. When the probe is far from the source branes, the volume of K3 becomes $V(r) \cong V_0 \gg V_*$. If the probe approaches to the singularity and r decreases, at the point $V(r) = V_* \Leftrightarrow r = r_e := \frac{2V_0}{V_0 - V_*} |r_2|$, the probe becomes tensionless, and moreover, when $r < r_e$ the tension becomes negative. This critical point r_e is called “enharon radius”. The r dependence of the volume of the K3 is shown in figure 2.

Negative tension is rather pathological. It seems that near the enharon radius, the classical supergravity is no more valid. Actually, when a nonperturbative object like a D-brane become light, even the perturbative string is no more valid. In the next section, we collect some observations, and mention the interpretation proposed by Johnson, Peet and Polchinski.

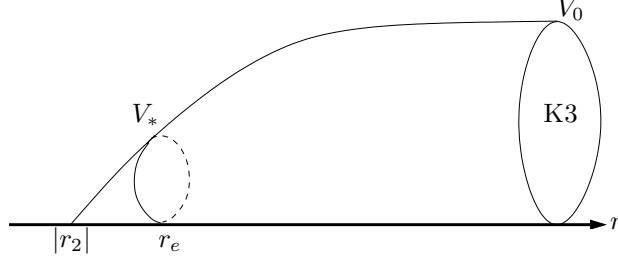


Figure 2: The image of the r dependence of the volume of the K3 $V(r)$. At large r , the volume is V_0 and we assume it is large. At $r = r_e$, the volume becomes V_* , where the stringy effect is relevant. At $r = |r_2|$, $V(r) = 0$.

3 Gauge symmetry enhancement and D6-D2* shell

3.1 type IIA string theory on K3 as a gauge Higgs system

In this section, we consider the 6-dimensional gauge theory obtained by the compactification of type IIA string theory on K3, and for a while, let us consider the background without D6-D2* branes.

First, we look at two U(1) gauge field of the following. One is obtained directly from ten dimensional $C^{(1)}$ by $C_m^{(1)}$ where $m = 0, 1, \dots, 5$. The other $\tilde{C}^{(1)}$ is obtained from ten dimensional 5-form $C^{(5)}$ (electromagnetic dual of 3-form $C^{(3)}$) by the relation $C^{(5)} = \tilde{C}^{(1)} \wedge \text{vol}(K3)$, where $\text{vol}(K3)$ is the volume form of the K3. Note that a D6-D2* has a magnetic charge $(1, -1)$ of $(C^{(1)}, \tilde{C}^{(1)})$.

Next, we have a scalar field $H = V - V_*$, where V is the volume of the K3. The vacuum expectation value of H is $V_0 - V_*$. Note that H is a moduli, which means the vacuum expectation value of H takes continuous values, and $\langle H \rangle$ parameterizes the vacuum.

Finally, there are a particles of D4-D0*. D4-D0* is the D4-brane wrapped on K3, and there appear the negative D0-brane charge for the same reason as the D6-D2* case. We show the D4-D0* configuration in table 2. We can see that it is actually a particle in 6-dimensions from the table 2.

	0	1	2	3	4	5	6	7	8	9
	$\mathbb{R}^{5,1}$						K3			
D4	○	×	×	×	×	×	○	○	○	○
D0*	○	×	×	×	×	×	×	×	×	×

Table 2: D4-D0* system

The action of the D4-D0* can be obtained by the same way as the D6-D2* case. The result is

$$S = - \int d\xi^0 \frac{\mu_4}{g_0} (V_0 - V_*) \sqrt{-g_{00}}.$$

We can read the mass of the D4-D0* as $\frac{\mu_4}{g_0} (V_0 - V_*)$, which is proportional to the $\langle H \rangle$. If V_0 is not much larger than V_* , then, D4-D0* particles is light, and easily pair created or pair annihilated. Therefore, we should treat D4-D0* particles as fields. Note that this field is electrically charged $(-1, 1)$ under the $C^{(1)}, \tilde{C}^{(1)}$ so the field is complex. Moreover, we assume this field is a vector field. We denote this field as W_m^- and its complex conjugate W_m^+ .

We summarize these actors in the table 3. This shows that the 6-dimensional system includes the $U(2)$ gauge theory with massless adjoint Higgs. The $U(2)$ gauge field is

$$A_m = \begin{pmatrix} C_m^{(1)} & W_m^+ \\ W_m^- & \tilde{C}_m^{(1)} \end{pmatrix}.$$

Actually, the charge of the D4-D0* is consistent with this picture.

origin	field	interpretation
10-d 1-form	$C^{(1)}$	photon
10-d 5-form	$\tilde{C}^{(1)}$	another photon
volume of K3	H	massless Higgs field
D4-D0*	W_m^\pm	W-boson

Table 3: The fields that we observe in 6-dimensional gauge theory obtained from type IIA theory compactified on K3. They are interpreted as fields in a Higgs mechanism.

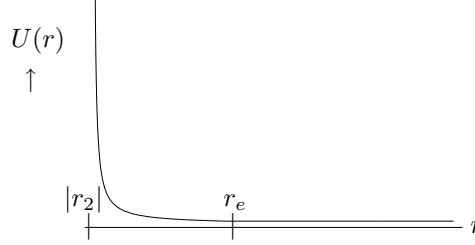


Figure 3: The graph of potential term in (5). $U(r) = -\frac{2\mu_6}{g_0}(Z_6^{-1}(r)V_0 - Z_2^{-1}(r)V_*)$. If $r > r_e$, the potential is flat. If $r < r_e$, there are repulsive force.

What is the role of the D6-D2* in this picture? It has magnetic charges of the photon. Hence, we can conclude the D6-D2* is the 'tHooft-Polyakov monopole in this gauge Higgs system. Near the center of the classical 'tHooft-Polyakov monopole, the value of Higgs becomes small and the W-boson mass becomes small.

When $H = 0$ the W-bosons become massless, and the gauge symmetry is enhanced from $U(1)^2$ to $U(2)$. At the enhançon radius of D6-D2* system, this gauge symmetry enhancement occurs. The name “enhançon” is from this gauge symmetry *enhancement*.

3.2 Enhançon as a shell

Let us go back to the D6-D2* system and consider the problem that the tension of the probe becomes negative. In the field theory, this is not a problem. In that case, only the mass square is appear in the theory, which mean $m_W^2 \propto \langle H^2 \rangle$.

In the probe theory, we can make a trick and avoid a negative mass as follows. We can rewrite the probe action (3) outside the enhançon radius as

$$S = - \int d^3\xi e^{-\Phi(r)} \mu_6 \sqrt{-(V(r) - V_*)^2 \det g_{ab}} + \dots \quad (5)$$

If we use this form also inside the enhançon radius, the mass does not become negative, but another problem occur. Inside the enhançon radius, the potential term of (5) is not constant. It becomes

$$S = \int d^3\xi \left[\frac{2\mu_6}{g_0} (Z_6^{-1}(r)V_0 - Z_2^{-1}(r)V_*) + (\text{higher order in } \dot{r}) \right]. \quad (6)$$

The potential is flat when $r > r_e$. On the other hand, when $r < r_e$, there are repulsive force. Therefore, D6-D2* probe cannot stop at a point inside the enhançon radius and the nearest point it can stop is the enhançon radius.

Now, let us consider how to construct the D6-D2* system and repulson singularity by gathering N D6-D2*'s. Let us observe one of the N D6-D2*'s, and others as sources of the background field, that is we separate D6-D2*'s to a probe brane and source branes. This is a kind of mean field approximation. The prove D6-D2* can be stops anywhere when it is apart enough from the source branes. Then, we move slowly the whole D6-D2*'s to a point to make the singularity. But when they are closer each other

than the enhançon radius, there are repulsive force and the configuration is unstable. In the result, the closest stable configuration is one in which the D6-D2*'s form a S^2 shell at the enhançon radius. The image of this observation is shown in figure 4

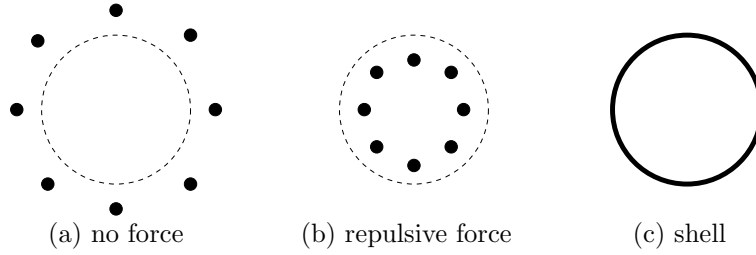


Figure 4: To make singularity by gathering the D6-D2*. The dashed circle expresses the enhançon radius. If D6-D2*'s are put as (a), the configuration is BPS and there is no force. On the other hand, if D6-D2*'s are more close each other and put as (b), there are repulsive force and the configuration is unstable. So the closest stable configuration is the shell as (c).

In the interior of the shell, it seems that the metric is flat because there are no source in the interior the shell. Consequently, there are no singularity in this configuration. This mechanism of resolution of singularity is called “enhançon mechanism”.

There is still another problem. We naively mention that D6-D2*'s are form the shell. However, if we treat D6-D2*'s as finite number of thin object, they cannot form a whole shell. More concretely, there are $SO(3)$ symmetry in the original D6-D2* system, but we cannot realize a $SO(3)$ invariant configuration by putting finite number of points on the S^2 . Therefore, on the enhançon shell, the branes are no more thin, but they seem to melt, mixed to each other and become a higher dimensional object. We review this picture from another point of view in the next section.

4 Enhançon and fuzzy sphere

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In order to see how the D6-D2* branes form a S^2 , let us go to a dual picture — the D3-D5 system. The D3-D5 system is, as shown in table 2, composed of 2 D5-branes and N D3-branes attached to both D5-branes. The two D5-branes correspond to the curvature of K3 in the D6-D2* system, and N D3-branes correspond to N D6-D2* branes in the D6-D2* system. Therefore the enhançon can be seen in the D3-D5 system as D3-brane shell. Let us see how the D3-branes form a shell.

From the point of view of the gauge theory on the D5-branes, the D3-branes are the magnetic monopole — 'tHooft Polyakov monopole. classical solution of 'tHooft Polyakov monopole is not a point like thin object, but a fat object. In brane picture, this is a tube like configuration as shown in figure 5.

	0	1	2	3	4	5	6	7	8	9
D3	○	×	×	×	○	○	○	×	×	×
D5	○	○	○	○	○	○	×	×	×	×

Table 4: D3-D5 system

Let us consider this system from the point of view of the gauge theory on the N D3-branes. The theory on the D3-branes is the super symmetric $U(N)$ gauge theory. It includes the gauge field A_μ , $\mu = 0, 4, 5, 6$ and adjoint scalar field ($N \times N$ hermitian matrix) X^I , $I = 1, 2, 3, 7, 8, 9$. The matrices X^I are interpreted as the coordinate of the position of the D3-branes. We use the following anzats here.

²This section was omitted in the talk.

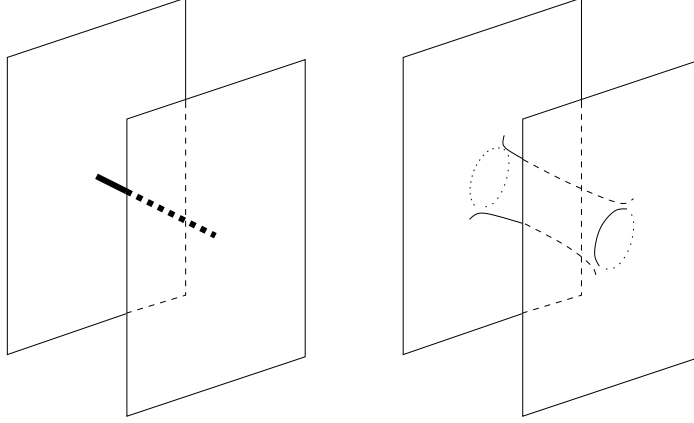


Figure 5: The image of the D3-D5 system. The D3-branes are attached to two D5-branes. From the point of view of the gauge theory on D5-branes, the D3-branes are smeared and become a tube. The smallest radius of the throat is to be enhançon radius [4, 7].

- Only three of the scalar X^1, X^2, X^3 have nontrivial value. The other field: the gauge field and X^7, X^8, X^9 is set to be 0.
- X^1, X^2, X^3 depend only on one of the space-like coordinate σ of the world volume.

Since we want BPS solutions, we should consider the Bogomolnyi equation instead of the equation of motion. If a configuration satisfies Bogomolnyi equation, that configuration also satisfies the equation of motion.

In the above ansatz, Bogomolnyi equation becomes

$$\partial_\sigma X^i = \frac{i}{2} \varepsilon^{ijk} [X^j, X^k]. \quad (7)$$

There are trivial solutions in which the X^i 's are independent of σ .

$$\partial_\sigma X^i = 0, \quad [X^i, X^j] = 0, \quad (i, j = 1, 2, 3).$$

In this case, X^i , $i = 1, 2, 3$ can be diagonalized at the same time.

$$X^i = \text{diag}(x_1^i, x_2^i, \dots, x_N^i). \quad (8)$$

This shows that the position of the a -th D3-brane is (x_a^1, x_a^2, x_a^3) . So this solution is N of thin D3-branes, which means that the position of each brane is sharply determined. The solution (8) is not the solution we need. In this solution, there are no D5-branes, and not the dual of original D6-D2* system.

We show here another solution. The shape of this solution is like the figure 6, and called “funnel solution” [8]. In that solution, the position of each branes are blurred and has $SO(3)$ symmetry. These are necessary properties of the enhançon, and we regard this solution as enhançon. The solution is

$$X^i(\sigma) = -\frac{1}{\sigma} \alpha^i,$$

where α^i is N -dimensional representation matrix of $SU(2)$ which satisfy

$$[\alpha^i, \alpha^j] = i \varepsilon^{ijk} \alpha^k.$$

In this solution, from the Casimir operator of N -dimensional representation, the following relation is satisfied.

$$(X^1)^2 + (X^2)^2 + (X^3)^2 = (R(\sigma))^2 \cdot 1_N, \quad R(\sigma)^2 = \frac{1}{\sigma^2} (N^2 - 1),$$

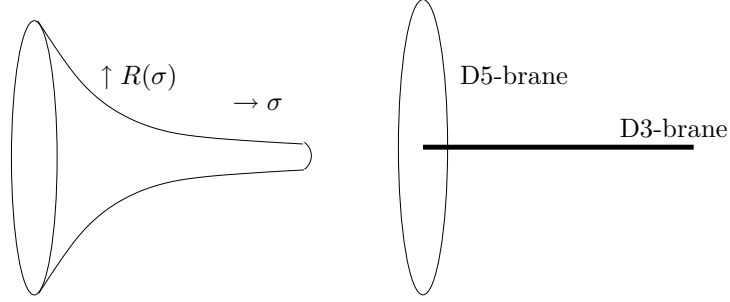


Figure 6: The image of the funnel solution. The section of constant σ is a S^2 of radius $R(\sigma) \propto \sigma^{-1}$. This solution can be interpreted as blurred picture of N D3-branes ending on a D5-brane. Actually it is known that this solution has a D5-brane charge.

where 1_N is the $N \times N$ identity matrix. As a result, if we consider the constant σ section, (X^1, X^2, X^3) is formally a S^2 of radius $R(\sigma)$. This space is called “fuzzy sphere” [10].

This geometry can be interpreted as the figure 6. The section of constant σ is a “ S^2 ” of radius $R(\sigma) \propto \sigma^{-1}$ and divergent at $\sigma = 0$. This can be interpreted as there are a D5-brane at $\sigma = 0$. So this is a picture of a D5-brane and N D3-branes end on the D5-brane from the point of view of the gauge theory on D3-branes. Actually, it is shown that this solution has a D5-brane charge [9].

We obtained a solution which contains a D5-brane and N D3-branes. However, strictly speaking, this is not the solution we need. In this solution, there is only one D5-brane and the gauge group on D5-brane is $U(1)$, and this solution is Dirac monopole in view of the gauge theory on the D5-brane. The solution we actually need is shown in figure 5. In this case, there are two D5-branes and the gauge group on D5-brane is $U(2)$. There are adjoint Higgs fields on the brane gauge theory, and the gauge group is broken to $U(1)^2$. Moreover, the D3-brane is a ’tHooft-Polyakov monopole from the viewpoint of D5-brane gauge theory. To obtain this type of solution is a future problem.

5 Conclusion

In this paper, we review the enhançon mechanism — a mechanism of resolution of repulson singularity of D6-D2* system. In this mechanism, the D6-D2* makes a shell at the enhançon, which is in exterior the singularity. Since there is no source in the interior of the shell, the inside of the shell is flat and nonsingular. This shell turns out to be a fuzzy sphere.

We mention the recent progress about enhançon here. In [11], other types of 3 dimensional gauge theory appear by introducing some kinds of orientifolds. The investigations from the view point of gauge theory on the brane are also done in [12–17]. In these papers, enhançon and the gauge theory on “fractional D-branes” are studied. Fractional D-branes are wrapped D-branes on vanishing cycles of some (for example orbifold) singularity and their D-brane charge is actually fractional. enhançons appear also in these fractional D-brane system. In [18, 19] enhançon and the black hole thermodynamics are investigated. In [20], the supergravity solution of enhançon shell and its stability are studied. In [21] more on consistency of enhançon are corrected and applied to some general case.

Acknowledgement

I would like to thank the participants of the associated seminars, especially, Yoshifumi Hyakutake, Akihiro Ishibashi, Hideo Kodama, Makoto Natsuume, Jiro Soda, Masa-aki Sakagami for usefull discussion.

The work of the author is supported in part by the JSPS Research Fellowships for Young Scientists.

References

- [1] J. Polchinski, *String Theory I, II* (Cambridge University Press 1998)

- [2] M. Natsuume, talk at this workshop, gr-qc/0108059.
- [3] J. Soda, talk at this workshop, gr-qc/0108046.
- [4] C. V. Johnson, A. W. Peet and J. Polchinski, Phys. Rev. D **61** (2000) 086001 [hep-th/9911161].
- [5] P. S. Aspinwall, hep-th/9611137.
- [6] M. Bershadsky, C. Vafa and V. Sadov, Nucl. Phys. B **463** (1996) 398 [hep-th/9510225].
- [7] C. V. Johnson, Phys. Rev. D **63** (2001) 065004 [hep-th/0004068].
- [8] N. R. Constable, R. C. Myers and O. Tafjord, Phys. Rev. D **61** (2000) 106009 [hep-th/9911136].
- [9] R. C. Myers, JHEP **9912** (1999) 022 [hep-th/9910053].
- [10] J. Madore, Class. Quant. Grav. **9** (1992) 69.
- [11] L. Jarv and C. V. Johnson, Phys. Rev. D **62** (2000) 126010 [hep-th/0002244].
- [12] N. Evans, C. V. Johnson and M. Petrini, JHEP **0010** (2000) 022 [hep-th/0008081].
- [13] M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda, R. Marotta and I. Pesando, JHEP **0102** (2001) 014 [hep-th/0011077].
- [14] M. Frau, A. Liccardo and R. Musto, Nucl. Phys. B **602** (2001) 39 [hep-th/0012035].
- [15] M. Petrini, R. Russo and A. Zaffaroni, Nucl. Phys. B **608** (2001) 145 [hep-th/0104026].
- [16] M. Billo, L. Gallot and A. Liccardo, hep-th/0105258.
- [17] P. Merlatti, hep-th/0108016.
- [18] C. V. Johnson and R. C. Myers, hep-th/0105159.
- [19] N. R. Constable, hep-th/0106038.
- [20] K. Maeda, T. Torii, M. Narita and S. Yahikozawa, hep-th/0107060.
- [21] C. V. Johnson, R. C. Myers, A. W. Peet and S. F. Ross, hep-th/0105077.